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ANALYTICAL MODEL OF TRANSIENT MISSILE BEHAVIOR BY MEANS OF ELECTRICAL NETWORK ANALOGIES

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FINAL REPORT

Written By

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The Pennsylvania State University
15 December 1962

Prepared For

United States Army Ordnance Missile Command
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ABSTRACT

The purpose of the investigation was to determine the feasibility of obtaining an electrical circuit whose behavior would be a satisfactory analog of the dynamics of a missile in flight. It was originally hoped that this could be done with passive circuit elements; however, it was found that due to severe non-linearity and unilateral coupling, it was not possible to establish a passive circuit to represent even a linear approximation of the missile system.

It was also hoped to find a mathematical procedure for analyzing the analog circuit which would allow analysis of the missile system by analogous responses. Due to the severity of the non-linearity this was not possible. Because of the nature and importance of the stability problem in non-linear systems, it was recommended that the analysis of the missile system be accomplished by utilizing an active electrical analog simulator arranged specifically for solution of the set of behavior equations representing missile dynamics including wind profile forcing functions. A study was made of the various devices which could be used to perform the circuit operations involved. This study lead to the belief that the most practical simulator would be composed of standard analog components.

SECTION I

INTRODUCTION

This report covers research conducted under Contract DA-36-034-ORD-3513RD between AOMC and The Pennsylvania State University. The work was done in the Department of Electrical Engineering.

1.1 Object

The object of the investigation was to determine to what degree it would be possible to utilize electric circuits as analog devices so arranged as to simulate dynamic missile behavior.

1.2 Need for the Study

In the ideal case the provision of an electrical analog of the missile system would consist of representing the parameters of the missile system in terms of electrical parameters. This would permit personnel trained primarily in electrical theory to analyze the missile system in terms of electrical parameters with which they are more acutely familiar. In utilizing the electrical analog the procedure would be to obtain a set of behavior equations for the electrical system. This would consist of a set of differential equations the solution of which would be the response of the electrical system. This response could then be

interpreted in terms of the parameters and response variables of the missile system yielding the desired dynamic behavior. The success of this procedure depends upon obtaining a useable mathematical solution of the behavior equations. Any solution which is not in closed form would be unsatisfactory because of the obscurity of the information desired. What is needed is a method which can be applied to the system of equations in a manner similar to the way in which the Laplace transform is applied in obtaining the solution to systems of linear equations.

In the absence of a satisfactory mathematical solution an alternative procedure would be to provide the physical electrical analog system and interpret its response in terms of the response of the missile system.

SECTION II

ANALYSIS

2.1 General Procedure

The first step in approaching the problem was to obtain from AOMC a set of equations representing the dynamic behavior of a missile system. These equations are shown on Page 5 of reference 15. Supplementary to this the limited literature search, references 2, 3, 4, 5, 6, 7, 12, 16, was conducted to determine whether these equations were presented in the best form and to gain familiarity with the general approach to the study of missile dynamics. The result of this revealed that the equations as presented in reference 15 are substantially complete and representative of a practical missile system. These equations were used as the basis for analysis throughout this report.

Having obtained a satisfactory set of equations, these were examined to establish the mechanical parameters involved and the appropriate variables normally employed in missile dynamics studies. A study was then made to determine an appropriate set of electrical parameters and variables representing a suitable analog system. Since no analog system is necessarily unique then many analogs may exist, it was necessary to defer using a specific system until a study could be made of the mathematical operations and hence of the electric

circuit operations involved in representing each term of each equation in the set of behavior equations.

Having established circuit operations involved in the analog system a study was then made of the various electrical devices and circuits capable of performing these operations.

Examination of the behavior equations taken from reference 15 as shown in section 2.2 shows that they are severly non-linear. In view of this it was decided to obtain a first approximation to these equations by applying a linearizing technique. An analog circuit was then developed from the linearized equations and was studied to determine the best choice of circuit variables and parameters. It was immediately discovered that the original equations exhibit the characteristic of unilateral coupling. This implies that the electrical analog circuit must therefore contain unilateral devices. It was also found that "isolation" is a circuit operation which must be provided.

After the study of the linear circuit was completed a new analog circuit was developed embodying all of the non-linear terms of the original set of equations. This circuit was studied and conclusions drawn, details of which are presented later.

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After the study of the linear circuit was completed a new analog circuit was developed embodying all of the non-linear terms of the original set of equations. This circuit was studied and conclusions drawn, details of which are presented later.

2.2 The Equations

Consider first the set of equations taken from reference 15 which are based on Figure 1 taken from the same reference.

The nominclature used in the discussion of these equations is as follows:

$d = \frac{1}{\eta} \frac{\partial C_a}{\partial \alpha} F \frac{\rho}{2} S^2$

η = the moment of inertia

C_a = the coefficient of lift

F = the diameter of the rocket

ρ = the density of the air

V = the geometrical velocity

θ = the path angle ($V \sphericalangle$ local vertical)

φ = the attitude angle (long. axis $\sphericalangle$ local vertical)

χ = gyro program angle (vertical at launch)

α = the angle of attack (axis $\sphericalangle$ flow direction)

α_w = the wind angle ($V \sphericalangle V_r$)

β = control deflection angle

ϕ = the angle at earth center between launch and present position

M = the mass of the rocket

g = gravitation

F = the thrust in direction of axis

X = the axial air force

N = the normal air force

R = the normal vane force

V_r = the velocity relative to surrounding air

a_o = the coefficient of linear terms on β

b_o = the coefficient of α terms in β

C_1 = the restoring moment per unit moment of inertia

C_2 = the control moment per unit moment of inertia

d = the damping moment coefficient per unit moment of inertia.

The forces in the direction of the missile velocity, divided by the mass of the missile give the equation

$$\dot{V} = \frac{F-X}{M} \cos(\alpha-\alpha_w) - \frac{N-R}{M} \sin(\alpha-\alpha_w) - g \cos \theta, \tag{2-1}$$

while the forces perpendicular to the velocity, divided by MV give the equation

$$\dot{\theta} + \dot{\phi} = \frac{F-X}{MV} \sin(\alpha-\alpha_w) + \frac{N+R}{MV} \cos(\alpha-\alpha_w) + \frac{g}{V} \sin \theta. \tag{2-2}$$

The moments about the center of gravity of the missile divided by the moment of inertia yield the equation

$$\ddot{\psi} + d\,\dot{\psi} + C_1\alpha + C_2\beta = 0 \tag{2-3}$$

The idealized control equation becomes

$$\beta = a_o(\psi + \phi - \chi) + a_1(\dot{\psi} + \dot{\phi} - \dot{\chi}) + b_o\alpha, \tag{2-4}$$

and from angular relations

$$a - a_w = \psi - \theta. \tag{2-5}$$

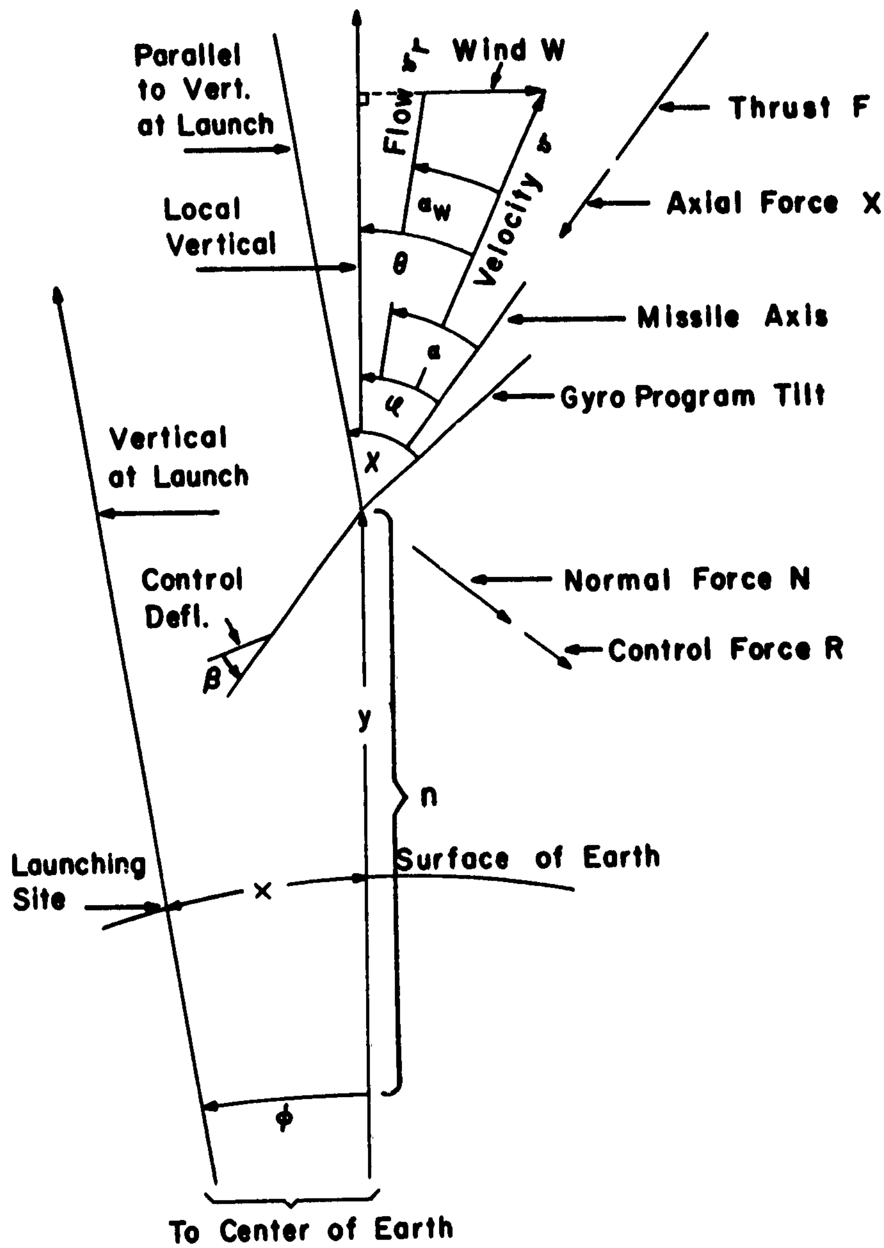

Parallel to Vert. at Launch
Flow
Wind W
Thrust F
Axial Force X
Local Vertical
Velocity
Missile Axis
Gyro Program Tilt
Vertical at Launch
Normal Force N
Control Defl.
Control Force R
y
n
Launching Site
x
Surface of Earth
To Center of Earth

Fig. I

It is realized that many of the parameters are not constant and therefore constitute either time dependent coefficients or non-linear terms, but for the present work it was decided to consider only the following five variables.

V = the velocity

α = the attack angle

ϕ = the center angle

θ = the path angle

ψ = the attitude angle

With the use of the relations found in reference 15, any one of these five variables can be removed by direct substitution. This leaves four variables and only three equations. A fourth equation can be derived using trigometric relationships from Fig. 1.

Since $V_x = V \sin \theta$, $V_y = V \cos \theta$, and $y = \int V_y dt$,

then $y = \int V \cos \theta \, dt$.

Now, $\chi = (R_o + y)\,\phi$

from which $V_x = \frac{d}{dt}\,(R_o + y)\,\phi = V \sin \theta$

which, when differentiated yields

$$V \sin \theta = (R_o + y)\,\dot{\phi} + \dot{y}\,\phi.$$

From which

$$V \sin \theta = \left[R_o + \int V \sin \theta \, dt\right]\dot{\phi} + V(\cos \theta)\,\phi \qquad (2\text{-}6)$$

Utilizing this fourth equation, we now have a set of four simultaneous integro-differential equations describing, to some degree of approximation, the dynamic response of a missile. If these equations were linear, well known techniques for handling linear sets could be applied and the entire missile behavior could readily be obtained.

Examination of the equations will reveal that they are decidely non-linear.

It is possible to solve these equations by numerical methods. However, such a procedure would best be done by employing a digital computer. Although this would yield a solution, this procedure would be contrary to the objective of this research. It was felt desirable to consider as a first approximation a linear model obtained by applying a linearizing technique to the equations.

2.3 The Linear Model

As a first approximation to the physical system, it was decided to linearize equations (2-1) through (2-5). This was done in order to provide the basis for a linear electrical network whose behavior would approximate the behavior of the missile.

Using the first term of the Maclaurin series for sine and cosine functions, equation (2-1) becomes

$$\dot{V} = -\frac{N+R}{M}\ (\alpha-\alpha_w) + \frac{F-X}{M} - g,$$

and equation (2-2) becomes

$$\dot{\theta} + \dot{\phi} = \frac{F-X}{MV}\ (\alpha-\alpha_w) + \frac{N+R}{MV} + \frac{g}{V}\,\theta.$$

Substituting the angular relation (2-5) in these equations yields

$$\dot{V} = -\frac{N+R}{M}\ (\varphi - \theta) + \frac{F-X}{M} - g, \tag{2-7}$$

and

$$\dot{\theta} + \dot{\phi} = \frac{F-X}{M}\ (\varphi - \theta) + \frac{N+R}{MV} + \frac{g}{V}\,\theta. \tag{2-8}$$

Substituting equations (2-4) and (2-5) into equation (2-3) and ignoring the higher derivative terms in (2-4), yields

$$\ddot{\varphi} + d\,\dot{\varphi} + C_1(\alpha_w + \varphi - \theta) + a_o C_2(\varphi + \phi - X)$$
$$+ b_o(a_w + \varphi - \theta) = 0. \tag{2-9}$$

For convenience, equations (2-6), (2-7) and (2-8) can be rewritten

$$\dot{V} = a_1(\varphi - \theta) = A_1 + A_2 \tag{2-10}$$

$$\dot{\theta} + \dot{\phi} - \frac{a_2}{V}\ (\varphi - \theta) - \frac{a_3}{V} - \frac{a_1}{V} = 0 \tag{2-11}$$

$$\ddot{\varphi} + a_4\,\dot{\varphi} + a_5(\varphi + \phi) + a_6(\varphi - \theta) = A_3 + A_4 \tag{2-12}$$

where

$$a_1 = \frac{N+R}{M}$$

$$a_2 = \frac{F-X}{M}$$
$$a_3 = g$$
$$a_4 = d$$
$$a_5 = C_2 a_o$$
$$a_6 = C_1 + C_2 b_o$$

are considered constant, and

$$A_1 = \frac{F-X}{M}$$
$$A_2 = g$$
$$A_3 = a_o C_2 \chi$$
$$A_4 = -(C_1 + b_o C_2)\alpha_w$$

can be considered forcing functions on the system. Equations (2-10) through (2-12) are still non-linear because of the existance of terms containing two variables. They can best be made linear by considering the missile velocity, V, to be a constant. This will yield results valid only over a short excursion of flight. While this is a serious restriction on the accuracy of the problem, it must be remembered that this linear set of equations is intended only as a first approximation.

With the velocity a constant, equations (2-9) through (2-11) become

$$a_1(\varphi - \theta) = A_1 + A_2 \tag{2-13}$$

$$\dot{\theta} + \dot{\phi} - a_2(\varphi - \theta) - a_3\,\theta = A_3 \tag{2-14}$$

$$\ddot{\varphi} + a_4\dot{\varphi} + a_5(\varphi + \phi) + a_6(\varphi - \theta) = A_4 + A_5. \tag{2-15}$$

These are linear differential equations in φ, θ, and ϕ, where:

$$a_1 = \frac{N+R}{M}$$

$$a_2 = \frac{F-X}{MV}$$

$$a_3 = \frac{g}{V}$$

$$a_4 = d$$

$$a_5 = C_2 a_o$$

$$a_6 = C_1 + C_2 b_o,$$

are considered constants, and

$$A_1 = \frac{F-X}{M}$$

$$A_2 = g$$

$$A_3 = \frac{N+R}{MV}$$

$$A_4 = a_o C_2 \chi$$

$$A_5 = -(C_1 + b_o C_2)\alpha_w$$

can be considered forcing functions on the system.

Based on the linearized equations (2-13) through (2-15), an effort was made to obtain a linear passive electrical analog circuit, having behavior equations of the same form. This

immediately led to difficulty because the coupling terms reveal a non-bilateral characteristic which is not attainable by passive linear circuit elements. This leads to the conclusion that there is no passive linear multi-mesh electrical network, whose behavior can be expressed by equations (2-13), (2-14) and (2-15). It should be noted that although these equations are linear, the coefficients of the coupling terms are not equal.

The unilateral coupling exhibited in these equations requires the introduction of unilateral devices into the analogous electrical network. The characteristics of such devices must be carefully defined. The unilateral device is symbolically shown in Figure 2. Here the device is represented as a rectangle having two input terminals and two output terminals. The ideal device is assumed to have infinite input impedance and zero output impedance and possesses no common ground. The capital letter in the upper center of the rectangle indicates the function being performed. For the linear model there are only three such functions, namely intergration represented by a capital I, differentiation represented by a capital D, and amplification represented by a capital A. The small letter in the lower center of the rectangle represents the scaler multiplier associated with the function. Since the device is a four terminal device with input and output terminal pairs isolated, it is possible for the output to be connected in either of two possible arrangements to provide the desired

polarity. The dot associated with input and output terminals indicates the instantaneous polarity. That is, when the input dotted terminal is driven positive the output dotted terminal will be positive.

It was initially decided that in the linear electrical analog circuit the various mechanical quantities should be represented by currents in which case the unilateral coupling will result in a potential introduced in one loop by virture of a current in a different loop. With this in mind, Figure 3 shows how the device can be used to provide unilateral coupling. Consider that I_1 is a loop current in loop 1 which passes through resistor R_1 and it is desired to induce a potential E_2 in loop No. 2 which is proportional to this current while at the same time prohibiting I_2 in loop 2 from inducing any potential in loop 1. The input terminals of the unilateral device are connected across R_1. Since the input impedance of the device is infinite, the device offers no loading on loop No. 1. The output voltage induced in loop 2 will then be described by the equation in the figure. It will be noted that the polarity marks indicate that the potential induced in loop 2 is positive for a positive current in loop 1. Had it been desired that this coupling be negative, it would only have been necessary to reverse the output terminals or the input terminals but not both.

The resistance R_1 as shown in Figure 3 not only provides the

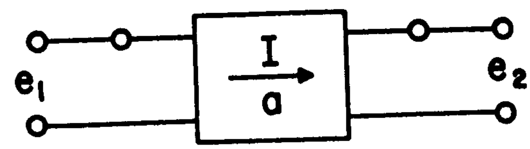


Fig.2

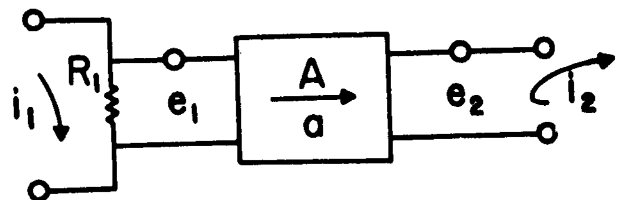


Fig.3

$$e_2 = ae_1 = aR_1 i_1$$

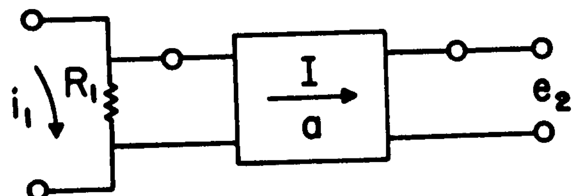


Fig.4

$$e_2 = aR_1 \int i_1 dt$$

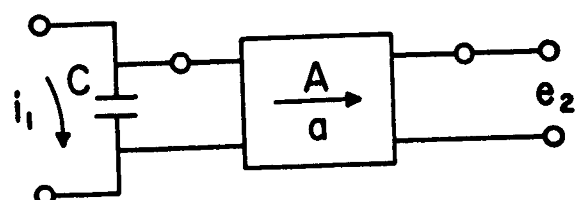


Fig.5

If $\frac{1}{c} = R_1$ in Fig. 3,

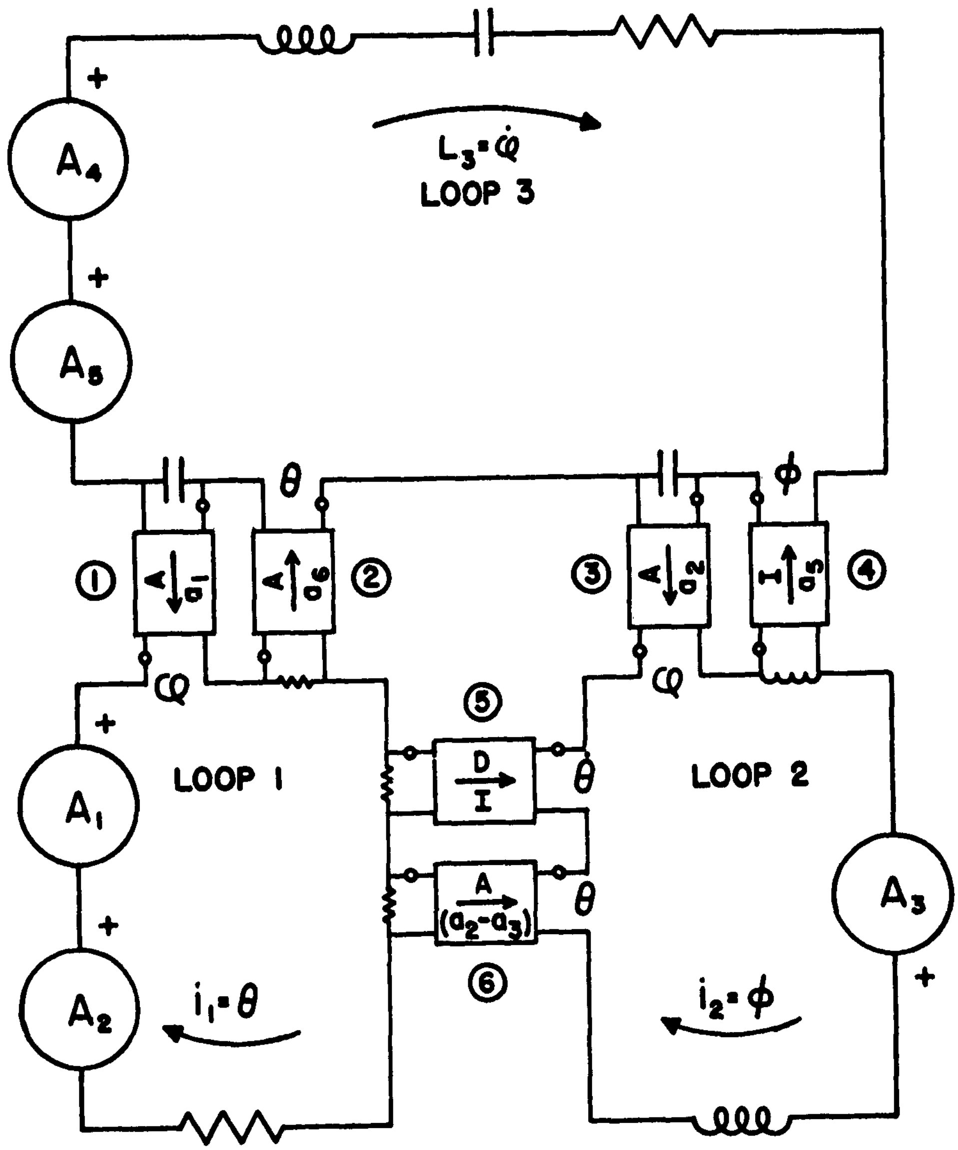
$$e_2 = \frac{a}{c} \int i_1 dt = aR_1 \int i_1 dt,$$

potential for unilateral coupling to loop 2 but also provides a self-impedance for loop 1. Since the function provided by the device is not always amplification but may be intergration or differentiation, Figures 4 and 5 show how the coupling could be arranged to provide potential in loop 2 which is the integral of the current in loop 1. It will be noted that this operation is not unique but that such an operation could be provided by either the circuit of Figure 4 or that of Figure 5. The difference between these two circuits however, lies in the self-impedance component which is desired in loop 1. In Figure 4 the self-impedance is resistance and the integration is performed by the device. In Figure 5 the self-impedance is that of the capacitor which integrates within itself, thus the device merely amplifies to provide the desired operation. It is evident that differentiation could be accomplished by using an inductor in loop 1 and permitting the device to simply amplify. On the other hand, if the self-impedance desired is resistance, then the device would be required to differentiate.

Using these basic operations, consider the circuit of Figure 6 and the three linear equations (2-13) through (2-15) which are presented by the three independent circuit loops. Loop No. 1 represents equation (2-13), loop No. 2 equation (2-14), and loop 3 equation (2-15). Let the variable in loop 1 be θ and be represented by I_1. Then the first term of equation (2-13) is represented by a potential drop across the resistance which

is the total resistance in the loop. The drive functions A_1 and A_2 are shown as generators, with polarities as indicated. The second term of equation (2-13) is a potential equal to a constant multiplier A_1 times the variable φ which is the integral of current I_3 in loop 3 representing $\dot{\varphi}$.
This potential is provided by unilateral device No. 1 shown at the left where the input potential taken across the capacitor in loop 3 is multiplied by the constant a_1 and is induced into loop 1 in the negative direction. Since the variable in loop 3 is $\dot{\varphi}$, φ is obtained by the integrating properties of the capacitor. It will be observed that there are no other unilateral devices feeding into loop 1 although there are three others feeding from loop 1 to other loops.

In equation (2-14), represented by loop 2, let it be observed that I_2 represents the variable ϕ even though ϕ itself does not appear in the equation. This choice was made for convenience in obtaining the necessary coupling terms to and from other loops. The first term of equation (2-14), $\dot{\phi}$, is the potential across the total inductance in loop 2. The second term $\dot{\Theta}$ is obtained by unilateral device No. 5 which differentiates the potential developed across a resistance in loop 1. The value of the resistance in loop 1 is not critical as long as the product of this value of resistance and the gain function of the unilateral device is equal to unity. Generally in such cases the resistance would be chosen



L3 = Q̇
LOOP 3
A4
A5
θ
ϕ
A
a1
A
a6
A
a2
I
a5
LOOP 1
LOOP 2
D
I
A
(a2-a3)
θ
θ
A1
A2
A3
i1 = θ
i2 = ϕ

FIGURE 6

as unity. The third term of equation (2-14) is a constant multiplier times θ which is obtained by unilateral device No. 6. Assuming the resistance in loop 1 is chosen as unity then the gain function of the unilateral device is set at the number $(a_2 - a_3)$. It will be noted that the polarity is arranged to provide a positive potential in loop 2 due to a positive current in loop 1.

The fourth term of equation (2-14) is a constant multiplier times the variable φ and should appear in the negative direction. This is obtained from unilateral device No. 3 which simply amplifies the potential taken across a capacitor in loop 3 producing a voltage in loop 2 equal to the number a_2 multiplied by the integral of $\dot{\varphi}$ which is φ. The drive function A_3 is again shown as a generator. Equation (2-15), represented by loop 3, can be analyzed in the same manner where it will be noted unilateral device No. 2 provides the fifth term and device No. 4 provides the fourth term.

The diagram shown in Figure 6 is not unique and other such diagrams could be arranged to represent the same set of equations. The one shown has been devised as an example to show the application of the unilateral devices and the flexibility in choice of variables represented by currents in the various loops.

The behavior equations of this circuit are of the form of equations (2-13) through (2-15), and in terms of the electrical

parameters are as follows:

$$R_1 i_1 - R_1 \int i_3 d_t = A_1 + A_2 \qquad (2\text{-}16)$$

$$L_2 i_2 + i_1 + (a_2 - a_3) i_1 - a_2 \int i_3 d_t = A_3 \qquad (2\text{-}17)$$

$$L_3 i_3 + R_3 i_3 + \frac{1}{C_3} \int i_3 d_t + a_5 i_2 + a_6 i_1 = A_4 + A_5 \qquad (2\text{-}18)$$

where:

$R_1 = a_1$ from equations (2-13) and (2-16)
$i_1 = \Theta$

$L_2 = 1$ from equations (2-14) and (2-17)
$i_2 = \phi$

$L_3 = 1$
$R_3 = a_4$
$C_3 = 1/(a_5 + a_6)$ from equations (2-15) and (2-18)
$i_3 = \dot{\varphi}$

This shows the relationship between the mechanical and electrical parameters.

Since the circuit of Figure 6 represents only a first approximation, it is now desirable to obtain a circuit which more accurately represents the missile system. This is accomplished by utilizing the non-linear equations without simplification.

2.4 The Non-Linear Model

If the mass, M along with ϕ, Θ, φ, and V, is allowed to vary,

equations (2-1), (2-2), (2-3) and (2-6) can be written:

$$\dot{V} = \frac{A}{M} \cos (\varphi - \theta) - \frac{B}{M} \sin (\varphi - \theta) - \cos \theta \tag{2-19}$$

$$\dot{\theta} + \dot{\phi} = \frac{A}{MV} \sin (\varphi - \theta) + \frac{B}{MV} \cos (\varphi - \theta) + \frac{C}{V} \sin \theta \tag{2-20}$$

$$\ddot{\varphi} + D\dot{\varphi} + E \quad - F\theta + G\phi + H\dot{\phi} = Q(t) \tag{2-21}$$

$$V \sin \theta = j\dot{\phi} \int_0^t V \cos \theta \, dt + V(\cos \theta)\phi \tag{2-22}$$

where:

$A = F - X$

$B = N + R$

$C = g$

$D = d + C_2a_1$

$E = C_1 + C_2a_o + C_2b_o$

$F = C_1 + b_o$

$G = C_2a_o$

$H = C_2a_1$

$J = R$

$$Q(t) \text{ Drive} = -(C_1 + C_2b_o)\,\alpha_w + C_2a_oX + C_2a_1\dot{X} \tag{2-23}$$

In the equation of the forcing function, $Q(t)$, $(C_1+C_2b_o)\alpha_w$ represents the wind, and $C_2a_oX + C_2a_1\dot{X}$ represents the pre-programmed gyro setting.

A four mesh electrical analog network was developed without benefit of any approximation beyond that already contained in the equations. This circuit is necessarily complicated because it contains all of the non-linear terms. Figure 7 should be compared with Figure 6.

In Figure 7 each rectangle represents a mathematical circuit operation. There are six basic operations performed by the following devices:

M = multiplier

R = resolver

A = amplifier

I = integrator

S = summer

G = inverter (reciprocal generation)

Each multiplier, designated M, has an output equal to the product of its two inputs. Each resolver, designated R, has two output terminals providing simultaneous sine and cosine functions of the input signal. The output of the summer, designated S, is the sum of its two input signals. The output of the inverter, designated G, is the inverse of its input. Finally the amplifier and integrator are shown with double letter notation where the upper letter represents the function (A in the case of amplifier and I in the case of integrator) and the lower letter represents the gain which can be either positive or negative. Since each of these

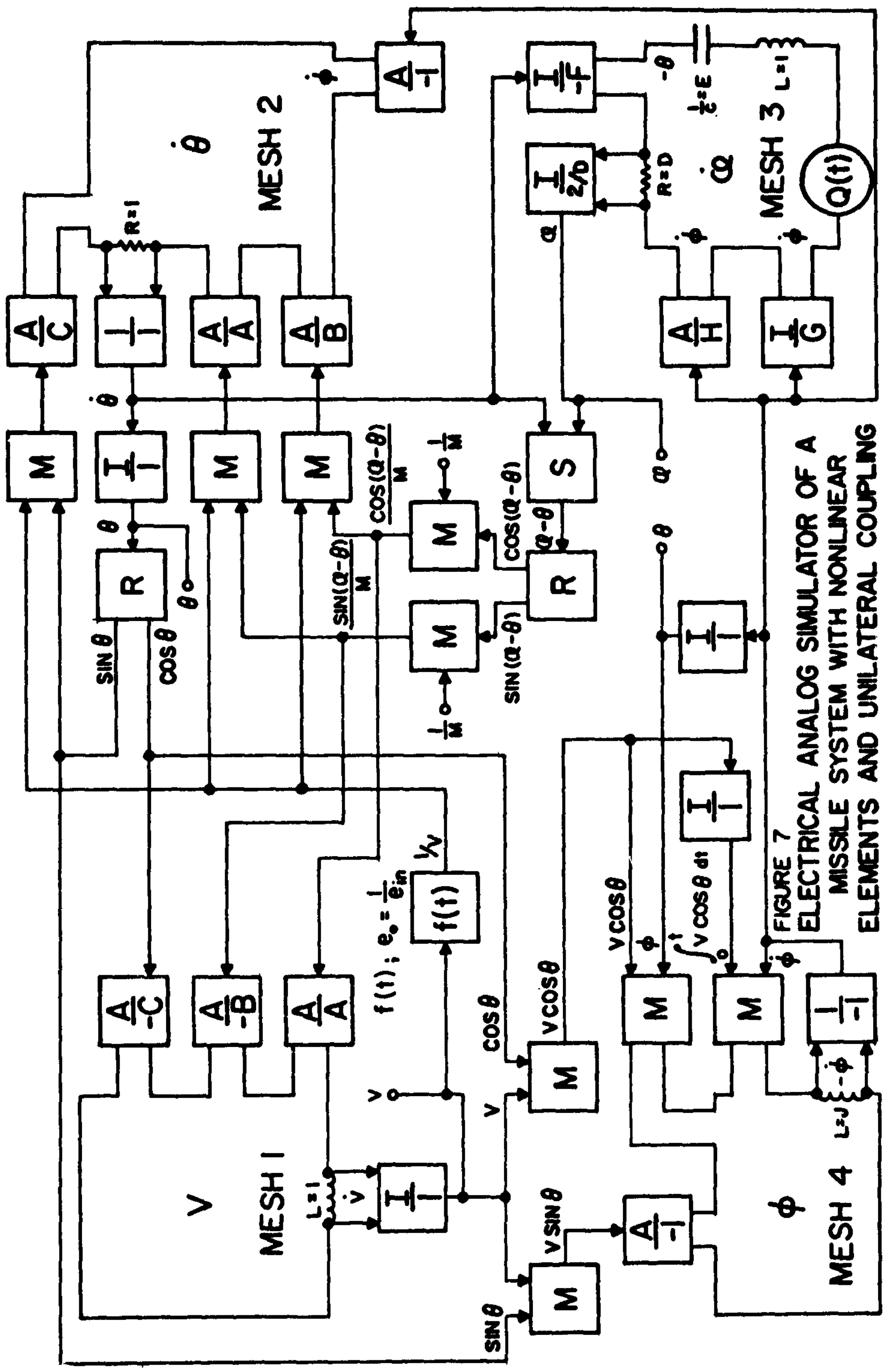


FIGURE 7
ELECTRICAL ANALOG SIMULATOR OF A MISSILE SYSTEM WITH NONLINEAR ELEMENTS AND UNILATERAL COUPLING

rectangles represents a device performing a mathematical operation it is assumed that a suitable device can be provided which will perform this operation to a satisfactory degree. Such devices will necessarily have to be unilateral within themselves, thus the arrowheads on the diagram show the direction of signal flow.

Basically the circuit of Figure 7 is a four mesh electrical network. These meshes are shown in the corners of the diagram and are numbered 1 through 4. They correspond to the variables in equations (2-19) through (2-22). For the sake of simplicity, the diagram is drawn as a single line diagram except at the points where signals leave or enter the meshes. For example, consider mesh 1 in which the variable is velocity. The only signal leaving the mesh is $\dot{V}$ which is the potential across the inductor fed to an integrator over two leads. The output of that integrator is a single lead feeding other loops. It is clear that the integration cancels the differentiation produced by the inductor. Hence, the output of the integrator is essentially the sum of the voltages entering the mesh. Although this could have been obtained by a simple summer, it was desired at this point to retain the identity of the mesh representing equation (2-19). It will be shown in section 3 that this is neither necessary nor desirable.

All voltages entering the mesh are shown entering in a series of double lead circuits from the amplifiers shown immediately to the right of the mesh. In each mesh the number associated

with the component (that is inductor, resistor, or capacitor) represents the coefficient in the respective equation. It should be possible now to follow the signals through the remainder of the diagram and to observe the generation of various non-linear and unilateral coupling terms.

It will be noted that the currents in meshes 1 and 4 represent the variables V and ϕ respectively, while in meshes 2 and 3 the currents represent the variables $\dot{\theta}$ and $\dot{i_\ell}$ respectively. The choice of currents representing derivatives in meshes 2 and 3 was made to avoid the use of differentiators because of the practical problems involved in their operation. Mesh 3 contains the drive on the system which is shown in equation (2-20). This is a potential generator representing wind angle and preprogrammed gyro setting. Each of the four variables can be read out at the appropriate terminal shown on the diagram as a small circle. Initial conditions are introduced on each of the integrators and as currents in loops 1, 3, and 4. The circuits providing these initial conditions are not shown because they are of secondary importance.

A careful study of this diagram will reveal the importance of each functional block and the part it plays in establishing the necessary intermesh coupling. It is expedient at this point to refer to Fig. 8 which shows a basic analog simulator diagram for the solution of the same 4 simultaneous equations. This simulator is comprised of standard analog computer com-

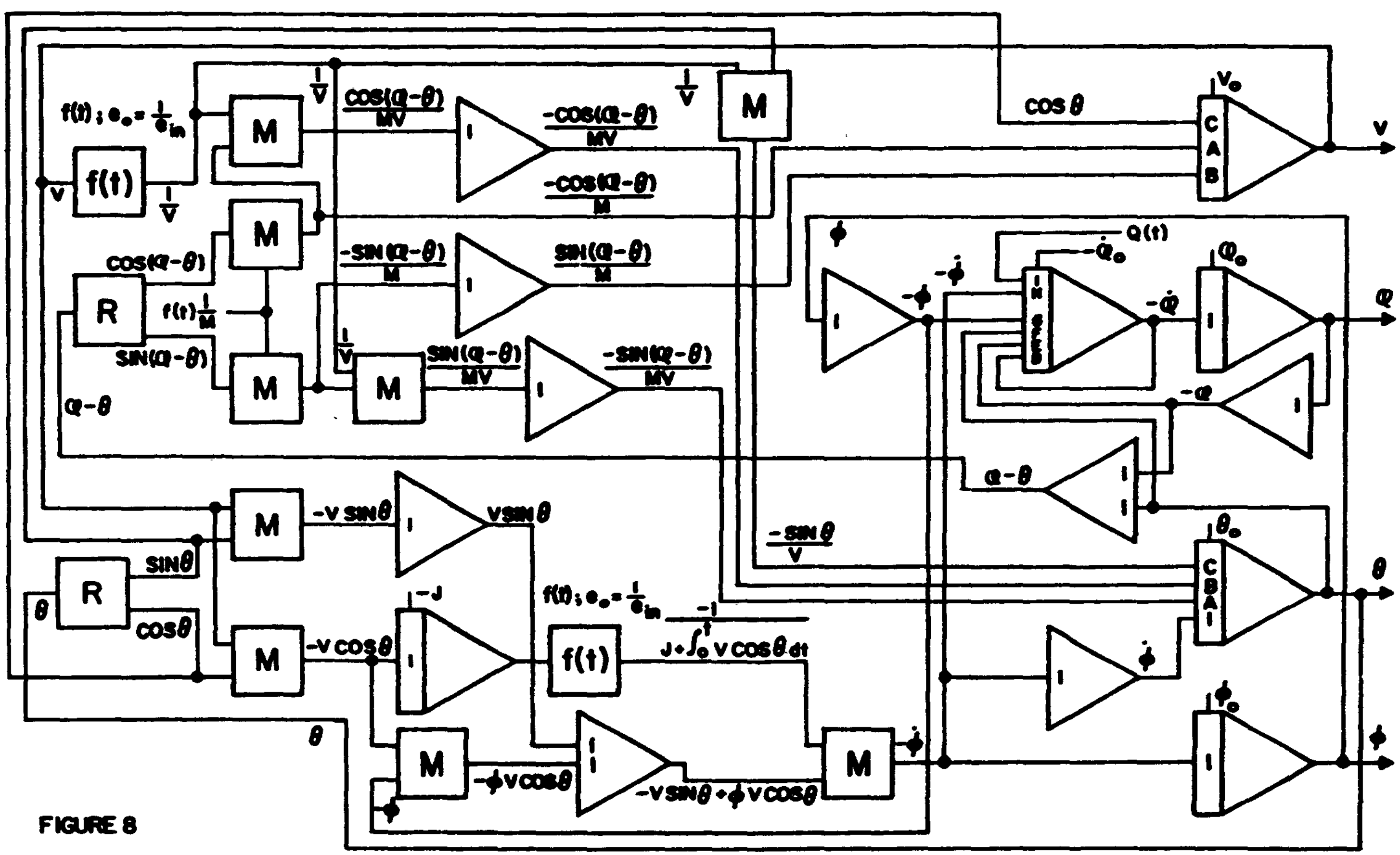


FIGURE 8

TYPICAL ANALOG FUNCTIONAL DIAGRAM REPRESENTING THE SYSTEM OF FIGURE 7

ponents and is introduced here for comparison. In reviewing Figure 7 the reader may gain the impression that the circuit is unduly complicated, however, comparison with Figure 8 will reveal that the simulator diagram is of approximately equal complexity. In Figure 7 there are 32 functional blocks, in Figure 8 there are 29 functional blocks. Considerable time was devoted to an attempt to reduce complexity of Figure 7 but with the support of the circuit of Figure 8 it was concluded that very little simplification is possible. It is evident that most of the diagram is comprised of functional blocks which are necessitated by the complex non-linear terms and unilateral coupling terms. This appears to an extent which obscures the basic four mesh concept. Figure 7 was prepared utilizing the well known transfer function notation representing the behavior of the individual devices. It is assumed, therefore, that each device performs the operation shown independent of the load on its output terminals. If this is not the case and loading must be considered, the determination of an analog network becomes many times more complicated, if indeed it is at all possible.

It should now be noted that although the circuit of Figure 7 is based on the four mesh concept where currents represent variables in the equations, all the functional blocks providing inter-mesh coupling are potential devices which have voltage input and output. It is not necessary or unique that the variables be represented by currents in the meshes. It

is possible to eliminate the meshes as such by replacing them by appropriate additional functional blocks. For example, mesh No. 1 could be eliminated along with its output integrator by simple summing of the voltages induced into this mesh and letting the output of that summer be the variable V. Similarly mesh 2 could be eliminated by summing all the voltages induced in that mesh and letting the output potential be θ. Meshes 3 and 4 could similarly be eliminated, however, the net results of this would be to provide a circuit diagram comprised completely of functional blocks, obscuring the concept of meshes. Such a diagram would reduce to that of Figure 8 where all functional blocks are performed by standard simulator elements. It is not surprising that this should happen since analysis of circuits of this nature has been the motivating influence leading to the development of the analog computer.

2.5 Stability Considerations

It was orginally desired to represent the missile by an electrical analog system for the purpose of organizing and simplifying the mathematical analysis involved.

It would be ideal if this analysis could be performed on a purely mathematical basis. This is possible in the linear case. Here the Laplace Transform is applicable and matrix methods exist to handle the simultaneous equations (2-13), (2-14) and (2-19). Unfortunately, the Laplace Transformation

is a linear transormation and is not applicable to problems of this type. Furthermore, the utility of the Laplace transformation is destroyed when the coefficients of the equations are functions of time.

There does not appear to be any uniform mathematical procedure for handling non-linear cases which would be the counterpart of the Laplace Transform for linear cases, references 1, 8, 9, 10. The state of the art in non-linear analysis does provide means of solving certain types of ordinary non-linear differential equations in closed form. In general, however, no procedure is known for obtaining closed form solutions. In most cases it is necessary to resort to iterative procedures which will yield approximations to the solutions. It is always possible to solve a non-linear differential equation by numerical methods. However, unless the problem is trivial the amount of work involved necessitates the use of a digital computer.

Solutions to simultaneous non-linear differential equations can sometimes be obtained by the process of elimination of variables. In some cases, however, the nature of the equations is not such that the elimination process is possible. Equations involved in this research are of this type. There appears to be no method other then to resort to numerical methods for solving such equations. It is common practice to analyze systems giving rise to equations of this type by employing analog or digital computers.

The analog computer is ideally suited to such analysis where extreme accuracies are not required.

One of the most important features of the solution of a non-linear system is the stability condition. The linear system is either stable or unstable. Stability is not determined by the drive function or initial conditions. In sharp contrast with this, however, a non-linear system may have regions of stability and regions of instability. In this case stability of the system may be a function of the initial conditions or the drive function. Stability criteria have been developed by Liapounoff, reference 9 and Poincaré, reference 10, but these apply only to non-linear systems with one degree of freedom. For this reason the analog computer is frequently used specifically for the study of stability of systems with multiple degrees of freedom. This is because the analog computer is itself a physical system so arranged as to have the characteristics of the system under study. The conditions under which the computer shows instability can be directly translated to conditions under which the system under study would similarly become unstable.

In order to establish the stability condition of the missile system it is necessary to obtain the best possible mathematical model of that system. This implies that alterations of the mathematical model by virtue of linearization or simplification or making approximations can result in erroneous information

concerning the stability condition. For this reason it is considered unwise to indulge in such simplification. It is therefore evident that any analog circuit or simulator representing the missile system must itself contain all necessary non-linear terms.

SECTION III

THE SIMULATOR CIRCUIT

3.1 Choice of Components

The choice of components comprising a simulator circuit depends primarily on the circuit operations to be performed. Those involved in this study were given in Section I and are repeated here for convenience:

Amplification

Summation

Integration

Inversion (reciprocal)

Multiplication

Function Generation (trigonometric)

The apparatus capable of performing each of these operations will be discussed separately.

3.1.1 Amplification. This is an operation on an input signal involving scalar multiplication. The output may be positive or negative and obeys the equation

$$\theta_o = \pm K\, \theta_1 \qquad (3\text{-}1)$$

where θ_o is a generalized coordinate representing the output quantity and θ_1 represents the input quantity. The scalar multiplier constant K may be greater, equal to, or less than unity. When K is less than unity the operation may be inter-

preted as attenuation.

A device used to perform amplification (amplifier) is intended to obey equation (3-1), however in practical cases it does so with limitations on frequency, amplitude, gain (K), and noise. An ideal amplifier would obey equation (3-1) without limitations and have infinite input impedance and zero output impedance. Practical amplifiers in general have the following properties:

a. Moderate input impedance

b. Moderate output impedance

c. Limited frequency response

d. Limited amplitude range

e. Variations in gain with signal level (nonlinearity)

f. Variations in gain with time

g. Drift (d.c. amplifiers only)

h. Noise in output

These properties give rise to departure from ideal performance. The evaluation of an amplifier depends upon the specifications dictated by the specific application. In any given case some properties may be more important than others. For use in an analog simulator properties c and d offer no problem. The important properties are b, e, f, g, and h. Such an amplifier must have:

1. Very low output impedance
2. Freedom from gain variations from any cause
3. Extremely small drift

4. Low noise output

It is well known that all of these characteristics can be obtained by the use of negative feedback provided the overall gain is low (in the order of 1 to 20). In order to provide isolation the output impedance must be very low. The use of negative feedback accomplishes this and at the same time renders the amplifier immune to influences which cause variation's in gain. Freedom from drift is accomplished by subsidiary stabilization circuits. Packaged amplifiers for this application are commercially available and are necessary to any analog circuit to be proposed.

3.1.2 Summation. This is readily accomplished by a passive circuit as shown in Figure 9 where the input and output quantities are voltage. The transfer function for this circuit is:

$$e_o = \sum_{j=1}^{n} \frac{R_p}{R_j} e_j \tag{3-2}$$

where R_p is the Thevenin resistance at the output terminals with the load resistance R_o connected. Equation (3-2) shows the summation operation and reveals the gain function to be a constant less than unity. The advantages of this circuit are its simplicity and reliability. One disadvantage lies in the fact that it is difficult to design when the values of gain for various inputs are different, that is, changing a resistance on one input affects the gain for all other inputs. The load resistance also affects the gain; hence, the circuit

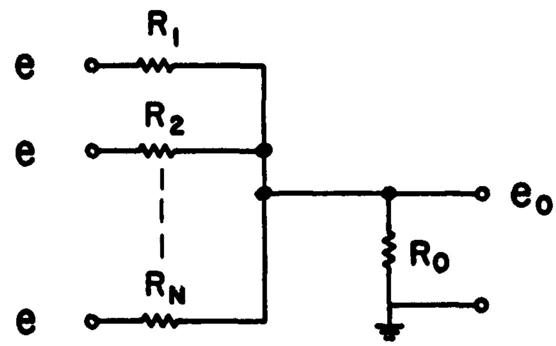


Fig.9. PASSIVE SUMMATION CIRCUIT

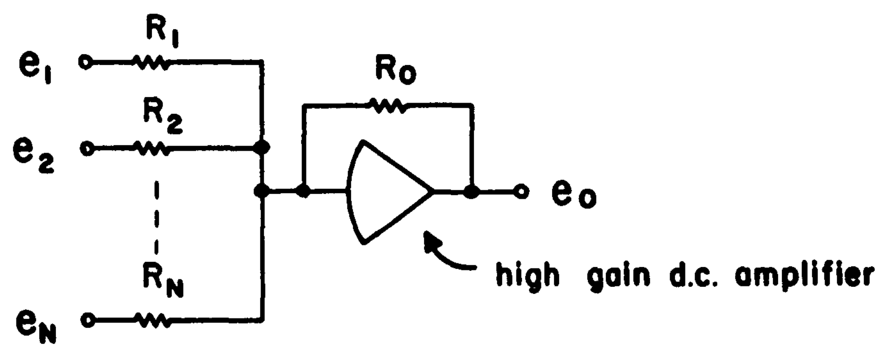


Fig. 10. IMPROVED SUMMATION CIRCUIT USING HIGH GAIN D.C. AMPLIFIER.

does not exhibit the desirable characteristic of isolation.

Presumably the attenuation resulting from multiple inputs in the case of the simple resistive network can be made up by following the summation network with a low gain stabilized amplifier. But if this is to be done, it is more expedient to incorporate the amplifier in the summation circuit as shown in Figure 10 where summation, amplification, and isolation are accomplished simultaneously yielding the further advantage that the gains for the various inputs are now independent as shown by the transfer function:

$$e_o = -\sum_{j=1}^{n} \frac{R_o}{R_j} e_j \tag{3-3}$$

This circuit is decidedly superior to the cascaded summation circuit and amplifier and is, therefore, commonly employed in analog computers.

3.1.3 Integration. A passive integrator is shown in Figure 11 working into a load resistance R_o. The output voltage is taken across R_o. To determine the transfer function of the circuit it is helpful to obtain the equivalent circuit of Figure 12 by applying Thevenin's theorem at the capacitor terminals. In this circuit e_T and R_T are the equivalent Thevenin values. In operational form the transfer function is:

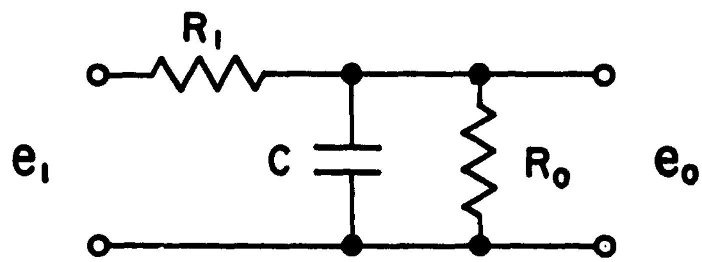


FIG 11 CIRCUIT OF PASSIVE INTEGRATIOR

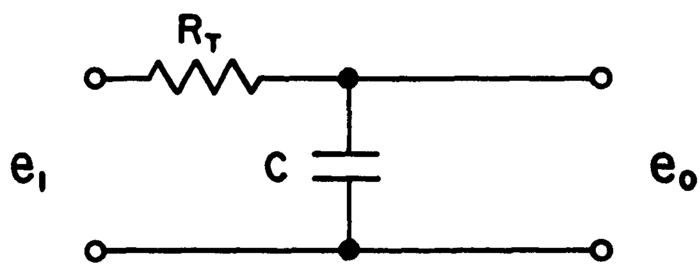


FIG 12 EQUIVALENT CIRCUIT OF PASSIVE INTEGRATOR

$$e_o = \frac{R_o}{R_1+R_o} \cdot \frac{1}{(1+R_T C p)} \cdot e_1 \qquad (3\text{-}4)$$

This equation shows that the circuit is not a true integrator because of the appearance of the "1" in the denominator. If the time constant $R_T C$ is made extremely large, equation (3-4) reduces to the approximation:

$$e_o = \frac{R_o}{R_1+R_o} \cdot \frac{1}{R_T C} \cdot \frac{e_1}{p} \qquad (3\text{-}5)$$

This reveals that to obtain good integration requires severe attenuation because of the large time constant.

Another possible passive integrator makes use of inductance in a circuit which is the dual of Figure 11. The output variable is then current rather than voltage. Such a circuit offers no advantage from an operational standpoint but possesses the disadvantage that a satisfactory inductor is extremely difficult to obtain because of non-linearity and hysteresis of the core material. The most satisfactory integrator is the type which consists of an operational amplifier in a standard analog integrator configuration. Even this is not a perfect integrator but it is many thousands of times superior to the best passive integrator. In this case the transfer function is:

$$e_o = \frac{1}{(1/G + R_1 CP)} \quad e_1 \qquad (3\text{-}6)$$

where G is the open loop gain of the amplifier and may be of

the order of magnitude of 10^5 or 10^6. In comparing equations (3-6) and (3-4) it can be seen that this obviates the difficulty with the passive circuit. Attenuation is not severe since it is no longer necessary to make the time constant large. In addition the output impedance is extremely low thus providing isolation.

3.1.4 Multiplication. This is one of the most difficult operations to perform by electrical means. Since the multiplier device has two inputs and one output, it is fundamentally a modulator. In a sense one input quantity modulates the other. No passive device is known which can perform this operation with the possible exception of a solid state semiconductor unit employing the Hall effect. Here the output voltage is the product of an input current and an input magnetic field according to the equation

$$e_o = K_h IB \tag{3-7}$$

where K_h is a constant determined by the physical configuration of the semiconductor element, I is an input current through the element, and B is the magnetic flux density normal to the element.

In practical cases I is measured in amperes and B in kilogausses. Even at these levels the output voltage is usually less than a volt because of the small value of K_h. One of the major difficulties encountered in building a

Hall multiplier is the problem of obtaining high magnetic flux density. This requires the use of a magnetic core exhibiting good linearity and low hysteresis.

In application the Hall multiplier would be arranged to have voltage input and output. Thus it would be necessary to have I proportional to e_1 (input No. 1) and B proportional to e_2 (input No. 2). Because of the low output voltage e_o it would be necessary to provide cascade amplifiers to bring e_o up to a useable level. The need for such amplification destroys any advantage of having a passive multiplier.

Other forms of solid state multipliers have been investigated but none have been found which are better than the Hall type. For example; it is possible to perform multiplication by resistance modulation. That is, a device whose output circuit is isolated from the input circuit and whose output resistance is inversely proportional to the input voltage e_1. The second input voltage e_2 is then connected across the output terminals and the resulting current is proportional to $e_1 e_2$. A practical unit based on this principle requires several active biasing circuits which again destroys the passive nature of the device.

A great deal of commercial development has been done on function multipliers and some satisfactory units are commercially available. These are all active devices; purely passive multipliers apparently do not exist.

3.1.5 Inversion. This operation is closely related to multiplication because it constitutes multiplication by a reciprocal. Passive devices for inversion are more difficult to provide than those for multiplication. This is true because the

$$\lim_{e_1 \to 0} e_o$$

does not exist for an inverter. If one establishes a lower bound on e_1, the operation requires a finite but high output for small input. Also the

$$\lim_{e_1 \to \infty} e_o = 0$$

must be satisfied. No known passive device is capable of performing this operation. It is evident that the resistance modulator mentioned in 3.1.4 might be applicable to this case provided the output resistance is made proportional to the input voltage e_1 and a constant voltage source is connected to the output terminals. The output current would then be proportional to the reciprocal of the input.

From a practical viewpoint a resistance modulator inverter would require numerous biasing circuits subject to critical adjustment. The modulator would consist of a light source focused on a photoconductive cell. The photoconductive cell is electrically isolated from the input circuit and its resistance is the output quantity. The natural characteristic

of a photoconductive cell shows a decreasing resistance for increasing light flux. In order to make the resistance increase with light flux the input to the cell must be biased to produce maximum light intensity with minimum input voltage. In addition, the natural non-linear characteristic of the photoconductive cell would require some form of physical linearizing technique. Such devices also exhibit a drift characteristic which is not necessarily serious when performing multiplication but probably would be serious when performing inversion. A small drift would represent a large percent change in input level for small values of e_1.

Inversion can be successfully accomplished by most commercial multipliers; in fact many have "multiply-divide" switches to allow either operation.

3.1.6 Function Generation. This is an operation involving the trigonometric functions "sine" and "cosine" where the function generator would be required to provide output voltages representing the sine and cosine of the input voltage over a range of plus or minus two quadrants (0 to 180° in either direction). It is possible to generate these functions by passive resistance diode networks. The procedure for the design of such networks is given in nearly all texts on analog computers. It will be found, however, that to cover two quadrants will require a separate generator for each quadrant with all outputs combined in a summation circuit.

The cosine generator will require biasing one input of the summation circuit in order to have unity output with zero input. The overall function generator circuit cannot be completely passive because two of the input voltages to the summation circuit will have to be injected in the negative polarity in order to provide subtraction. Changing sign cannot be done passively and requires an operational amplifier. In this event it will be simpler to combine the operational amplifier with the resistance-diode network to produce an active function generator requiring fewer component parts. Such generators are very reliable and once adjusted to produce the desired function do not require frequent readjustment.

The need for function generators of the type under discussion is such a common occurrance in the field of analog analysis that complete packaged units are commercially available and require only an initial adjustment in order to obtain the desired function. In the case at hand one unit would be required for the sine function and a separate unit for the cosine function. The accuracy with which the function can be generated depends upon the number of segments used and can be made quite adequate for use in the simulator under consideration.

3.2 Choice of Circuit

3.2.1 Apparatus. It was shown in Section II that, because of the unilateral coupling exhibited by equations (3-1) through (3-3), it is not possible to simulate the missile system by a purely passive electrical circuit. The unilateral coupling terms require the employment of active isolation devices. The electronic amplifier with inverse feedback (operational amplifier) is ideally suited to this application. The isolation amplifier must satisfy all the requirements outlined in Section 3.1.1.

Figure 7, which shows the functional arrangement of the simulator, reveals that many amplifiers will be needed. In this case it would be desirable to have all amplifiers alike and interchangeable so that one or two spare amplifiers will suffice for maintenance. This in turn suggests an analog operational amplifier capable of operating at zero frequency. Commercial units with chopper stabilization are available from many manufacturers. Even if the circuit of Figure 5 were to be used to represent the missile system, it would be necessary to use six such operational amplifiers.

Now that it has been established that amplifiers are necessary for the purpose of providing isolation it is clear that they can simultaneously be used for amplification wherever necessary. It is now expedient to consider the three operations: summation,

integration, and function generation. These are the operations which can be performed by passive circuits. However, as shown in Section 3.1 these circuits do not possess the property of isolation and, therefore, must be cascaded with amplifiers for that purpose. It is better circuit design and performance is superior when the passive circuit is combined with the amplifier as outlined in any standard text on analog computing circuits.

In considering multipliers and dividers there appears to be no choice but to resort to the use of commercial units which are designed specifically for performing these operations with a high degree of accuracy.

3.2.2 Theoretical Considerations. It was shown in Section 2 that because of the nature of the equations involved, it is not possible to establish a simple straight forward mathematical procedure for analyzing the missile system in terms of the analog system. This is primarily because the state of the art in non-linear analysis does not provide for this possibility. Although the analysis of linear systems has been highly developed and many mathematical procedures such as Laplace Transform are readily available, the same is not true with non-linear systems particularly with more than one degree of freedom. There does not even appear to be a single procedure for the solution of non-linear equations representing one degree of freedom. The situation is further complicated by

the fact that the stability problem in non-linear analysis is much more difficult than for linear systems. Although stability criteria have been developed for special cases there is no general stability criteria for non-linear systems as such.

In the absence of a mathematical procedure it then becomes important that the analog circuit allow analysis of the system in terms of the response of the analog system. This is a common and effective method of engineering analysis. In the light of this it is important that the analog circuit be flexible to permit modifications to coincide with possible modifications of the behavior equations. It is also important that the circuit be such that it is convenient for setting coefficients and introducing initial conditions. One very important factor is that it must permit use of functions for which equations may not be known. In the case at hand this would be wind profile drives. This can readily be accomplished in an analog circuit by use of a standard curve follower device which is commercially available.

An analog circuit arranged to accurately represent the physical system will permit a study of the stability condition by observation rather than by theoretical means. The information gained by this procedure is an extremely important part of the analysis; in fact, it is because of this that approximations of the original equations could not be made.

3.2.3 Recommended Circuit. In view of the discussion covered in Sec. 3.1.1 and 3.1.2 it is recommended that the circuit of Figure 8 be used for the analog of the missile system. This circuit is comprised of standard commercial components all of which possess the property of isolation. It affords ease in setting coefficients and insertion of initial conditions. It provides access for reading out all important variables and input points for insertion of wind profile drive functions. It allows easy modification, should this become necessary, and will perform with satisfactory accuracy and reliability.

This circuit was chosen because it affords simplicity of operation and maintenance. Other circuits were considered but none were simpler or contained fewer circuit components.

3.2.4 List of Components.

Item No.	Description	Quantity
1	Operational Amplifier	20
2	Function Multipliers	12
3	Curve Followers	4
4	Function Generators	6

This list covers all needed functional devices allowing for modification and spares. In addition to these items it will be necessary to furnish a cabinet, circuit junction board, and other minor accessories according to the wishes of the group using the simulator.

It may appear that, since the components listed are commonly used on analog computers, it would be possible to purchase an analog computer complete as such. This may be possible; however, the array of devices listed is not usually found on a standard machine. Further a removable patch board is not essential. This decision should be made by appropriate personnel at AOMC.

SECTION IV

CONCLUSIONS AND RECOMMENDATIONS

4.1 Conclusions

It is concluded from this research that:

a. It was not found possible to represent the missile system by a completely passive electrical analog circuit.

b. Due to the severe non-linear character of the set of behavior equations used to describe missile dynamics, it is not possible to find a mathematical procedure for analysis which would be the counterpart of the Laplace Transform for linear analysis.

c. Because of the importance of the stability problem in non-linear analysis, it is essential to use the most accurate mathematical model possible for the study of missile dynamics. Thus; linearization or other simplifying techniques should not be applied to the mathematical model.

d. The term "indicial admittance" is a property of a linear system and has significance insofar as it is used in conjunction with the superposition integral or other linear analytic techniques. Since superposition does not apply to non-linear systems, indicial admittance as such does not have

equivalent significance for non-linear systems and, therefore, for the missile system.

<u>e</u>. Analysis of missile dynamics can probably best be accomplished utilizing an analog simulator and interpreting the missile response in terms of the simulator response. This amounts to using the simulator to bypass the mathematical difficulties arising out of the required non-linear analysis.

4.2 Recommendations

As a result of the research covered by this report the following recommendations are offered:

<u>a</u>. An analog simulator should be built or purchased as outlined in Section 3.2.3.

<u>b</u>. Any contemplated program for the development of special analog devices should be abandoned. This is because the product of such development does not promise to be superior to the circuit recommended, either from the standpoint of circuit performance or information it would reveal to personnel.

<u>c</u>. Further research should be carried out directed toward development of useful mathematical techniques for handling systems of non-linear differential equations, preferably of the type encountered in missile studies. Such research should be conducted by a group consisting of applied mathematicians and engineers and should be planned as a long term program.

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(b) Authors - Carl Volz and William S. Adams
(c) Date - 15 December 1962
(d) Final Technical Report - (Unclassified)
(e) Department of Army Project Number - 516-01-004
(f) Contractor - The Pennsylvania State University
(g) Contract Number - DA-36-034-ORD-3513RD
(h) Agency - United States Army Ordnance Missile Command, Redstone Arsenal, Alabama
(i) Abstract - (Unclassified)

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It was also hoped to find a mathematical procedure for analyzing the analog circuit which would allow analysis of the missile system by analogous responses. Due to the severity of the non-linearity this was not possible. Because of the nature and importance of the stability problem in non-linear systems, it was recommended that the analysis of the missile system be accomplished by utilizing an active electrical analog simulator arranged specifically for solution of the set of behavior equations representing missile dynamics including wind profile forcing functions. A study was made of the various devices which could be used to perform the circuit operations involved. This study lead to the belief that the most practical simulator would be composed of standard analog components.

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(a) Title - Analytical Model of Transient Missile Behavior by Means of Electrical Network Analogies - (Unclassified)
(b) Authors - Carl Volz and William S. Adams
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Abstract

The purpose of the investigation was to determine the feasibility of obtaining an electrical circuit whose behavior would be a satisfactory analog of the dynamics of a missile in flight. It was originally hoped that this could be done with passive circuit elements; however, it was found that due to severe non-linearity and unilateral coupling, it was not possible to establish a passive circuit to represent even a linear approximation of the missile system.

It was also hoped to find a mathematical procedure for analyzing the analog circuit which would allow analysis of the missile system by analogous responses. Due to the severity of the non-linearity this was not possible. Because of the nature and importance of the stability problem in non-linear systems, it was recommended that the analysis of the missile system be accomplished by utilizing an active electrical analog simulator arranged specifically for solution of the set of behavior equations representing missile dynamics including wind profile forcing functions. A study was made of the various devices which could be used to perform the circuit operations involved. This study lead to the belief that the most practical simulator would be composed of standard analog components.